國立中央大學八十七學年度轉學生入學試題卷

數學系　三年級　　　科目：高等微積分　　　共 1 頁 第 1 頁

一、簡答題：請寫出理由、例子、反例、或反証，每題十分。

1. Is the function $f(x) = \frac{1}{x}$ uniformly continuous on $[1, \infty)$?

2. Evaluate the limit $\lim_{x \to 0^+} (\tan x)^x$.

3. Must every Cauchy sequence $\{a_n\} \subseteq \mathbb{R}$ be bounded?

4. Let $f : \mathbb{R}^n \mapsto \mathbb{R}^m$ be a continuous function and $A \subseteq \mathbb{R}^n$ be bounded. Is $f(A)$ bounded in $\mathbb{R}^m$?

5. Each f_n is a polynomial and $\{f_n\}$ converges uniformly to f on $[0, 1]$. Must f be a polynomial?

6. Is any linear map $f : \mathbb{R}^n \mapsto \mathbb{R}^m$ continuous?

二、證明題，每題二十分。

1. Suppose $\sum_{n=0}^{\infty} c_n x^n$ converges at $x = R$, and $0 < \epsilon < R$. Show that $\sum_{n=0}^{\infty} c_n x^n$ converges uniformly on $[-R + \epsilon, R - \epsilon]$.

2. Prove that we can solve $\begin{cases} xu + yv^2 = 0 \\ xv^3 + y^2u^6 = 0 \end{cases}$ uniquely for (u, v) as functions of (x, y) near $(x, y, u, v) = (1, -1, 1, -1)$, and compute the values $\frac{\partial u}{\partial x}(1, -1)$, $\frac{\partial u}{\partial y}(1, -1)$, $\frac{\partial v}{\partial x}(1, -1)$, and $\frac{\partial v}{\partial y}(1, -1)$.

參考用